[11:00am-11:35am] Overview of mini project and signal to noise ratio.

Write a narrative report (4-5 pages, not counting figures) The audience is other students in the class. Discuss the topics you've learning in the class and explain how they relate to the project (e.g. Fourier series).

Signal to noise ratio

$$\text{Signal to noise ratio} = \text{SNR} = \frac{\text{Signal power}}{\text{Noise power}}$$

Where the power is proportional to the squared amplitude of the signal (e.g. Watts). In decibels:

$$\text{SNR}_{dB} = 10 \log_{10}(\text{SNR})$$

The output $y(t)$ of a system $\mathcal{T}$ will often to contain a desired component, or the "signal" $x(t)$, and an undesired or "noise" component $v(t)$

$$y(t) = x(t) + n(t)$$

$$v(t) = y(t) - x(t)$$

$$SNR = \frac{\text{Average power } \{x(t)\}}{\text{Average power } \{v(t)\}} \frac{\text{Average power } \{x(t)\}}{\text{Average power } \{y(t) - x(t)\}}$$

When $n(t) = 0$, the SNR is infinity dB. When the signal has the same amount of power as the noise, the SNR is zero dB.

The amplitude of a digital audio signal represents a sampled and quantized voltage. Since the power is proportional to the voltage squared, the average power of a discrete signal with length N is

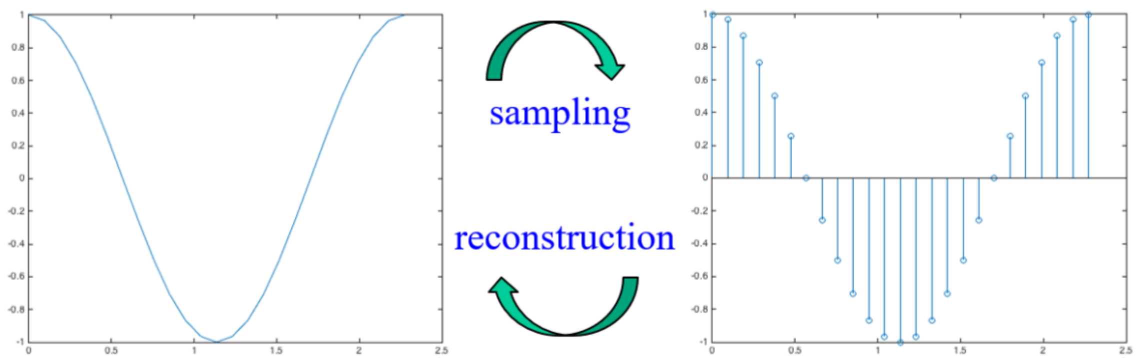
$$\text{Average power}\{x[n]\} = \frac{1}{N} \text{Total power } \{x[n]\} = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

$$\text{SNR} = \frac{\sum_{n=0}^{N-1} x^2[n]}{\sum_{n=0}^{N-1} v^2[n]} = \frac{\text{Average power}\{x[n]\}}{\text{Mean squared error}\{x[n], y[n]\}}$$

[11:35-12:15] Sampling and aliasing

Sampling it the conversion from a continuous-time signal to a discrete-time signal. Reconstruction is the reverse process. In the ideal case, it's possible to sample and perfectly reconstruct a signal. In practice, the reconstructed signal may be different.

Example: 440Hz cosine sampled at 10460 Hz (24 samples per period, 12× oversampled)



The sampling theorem says that if $f_s > 2f_{\max}$, it's possible to reconstruct a continuous signal $x(t)$ from its samples. If a signal contains frequencies higher than $\frac{1}{2}f_s$ prior to sampling, they will alias down to a frequency lower than $\frac{1}{2}f_s$. To guarantee this condition is met, we can apply a low-pass filter prior to sampling.

To convert from continuous time signal $x(t)$ to discrete-time signal $x[n]$, we sample at equally spaced points separate by the sampling period T_s.

$$x[n] = x(nT_s) = x\left(\frac{n}{f_s}\right)$$

The reconstruction process fills in the values between samples (interpolation). There are several options for interpolation:

- Draw straight lines between samples (linear interpolation)
- Fit a curve (e.g. bicubic interpolation)
- Another option is to hold a constant value until the next sample (leading to a stair-case approximation)

Example: sampling a sinusoid $x(t) = A\cos(\omega t + \phi)$

$$x[n] = x(nT_s) = A\cos(\omega n T_s + \phi) = A\cos(\hat{\omega} n + \phi), \qquad \text{where } \hat{\omega} = \omega T_s = 2\pi \frac{f}{f_s}$$

Application	Sampling Rate	Continuous-Time Frequency	Discrete-Time Frequency
Speech	8000 Hz	220 Hz	2 π 11/400 = 0.1728
Audio	44100 Hz	441 Hz	2 π 1/100 = 0.0628
Audio	48000 Hz	1320 Hz	2 π 11/400 = 0.1728

The conversion from discrete-to-continuous is dependent on the sampling rate—a cosine with discrete-time frequency of 0.1728 can be interpreted as a 220 Hz signal by assuming a sampling rate of 8000 Hz, and can be interpreted as a 1320 Hz signal by assuming a sampling rate of 48,000 Hz.

Aliasing occurs when a frequency above $f_s/2$ is mapped to a frequency below $f_s/2$ during the sampling process.

Example: Samples of a 2.4 Hz cosine are identical to samples of 0.4 Hz cosine